Submitted to *ApJ Letters*, October 1996

Effect of Gravitational Lensing on Measurements of the Sunyaev-Zel'dovich Effect

Abraham Loeb[1,3] and Alexandre Refregier[2,4]

1. Astronomy Department, Harvard University, 60 Garden St., Cambridge, MA 02138
2. Columbia Astrophysics Laboratory, 538 W. 120th Street, New York, NY 10027

ABSTRACT

The Sunyaev–Zel'dovich (SZ) effect of a cluster of galaxies is usually measured after background radio sources are removed from the cluster field. Gravitational lensing by the cluster potential leads to a systematic deficit in the residual intensity of unresolved sources behind the cluster core relative to a control field far from the cluster center. As a result, the measured decrement in the Rayleigh–Jeans temperature of the cosmic microwave background is overestimated. We calculate the associated systematic bias which is inevitably introduced into measurements of the Hubble constant using the SZ effect. For the cluster A2218, we find that observations at 15 GHz with a beam radius of $0'.4$ and a source removal threshold of 100μJy underestimate the Hubble constant by 6–10%. If the profile of the gas pressure declines more steeply with radius than that of the dark matter density, then the ratio of lensing to SZ decrements increases towards the outer part of the cluster.

Subject headings: cosmic microwave background – diffuse radiation – galaxies: clusters: general, individual (A2218) – gravitational lensing

1. Introduction

The Sunyaev–Zel'dovich (SZ) effect describes the distortion introduced to the Cosmic Microwave Background (CMB) spectrum due to its Compton scattering off free electrons, which are either hot (the *thermal* effect) or possess a bulk peculiar velocity (the *kinematic* effect; see reviews of both effects in Sunyaev & Zel'dovich 1980 and Rephaeli 1995). The thermal SZ effect provides an important diagnostic of the hot gas in clusters of galaxies, and by now has been

[3]email:aloeb@cfa.harvard.edu

[4]email:refreg@odyssey.phys.columbia.edu; also at the Department of Physics, Columbia University

measured in a number of systems (see Table 1 in Rephaeli 1995). The kinematic effect has an amplitude which is typically an order-of magnitude smaller and has not yet been definitively detected (see Rephaeli & Lahav 1991 and Haehnelt & Tegmark 1995, regarding prospects for a future detection).

It has long been realized that a measurement of the thermal SZ effect, combined with X–ray observations, can be used to estimate the distance to the cluster and hence the Hubble constant, H_0, under the assumption that the cluster is spherical (Cavaliere, Danese, & De Zotti 1977, Gunn 1978, Silk & White 1978, Birkinshaw 1979). The inferred value of the Hubble constant is inversely proportional to the square of the SZ temperature decrement. This approach had led to values of the Hubble constant which are typically on the low side of the range inferred from other methods (see, e.g., Table 2 in Rephaeli 1995). An often cited systematic effect that could account for this bias is elongation of the selected clusters along the line-of-sight. In this *Letter*, we explore a different effect which leads to a systematic bias towards low–H_0 values even if these clusters are perfectly spherical. The effect results from gravitational lensing by the cluster potentials.

Measurements of the decrement in the Rayleigh–Jeans (RJ) temperature of the microwave background due to the thermal SZ effect are routinely accompanied by the removal of background radio sources down to some flux threshold (see, e.g., Birkinshaw, Hughes, & Arnaud 1991). In this process, it is implicitly assumed that the flux threshold for the removal of sources behind the cluster core is the same as in a control field far from the cluster center. However, this assumption is not strictly true due to the inevitable magnification bias which is introduced by the gravitational lensing effect of the cluster potential. In reality, the cluster acts as a lens which magnifies and thus resolves sources that are otherwise below the detection threshold. The residual intensity of unresolved sources is therefore systematically lower behind the cluster core, as compared to that in the control field. Lensing artificially increases the flux deficit behind the cluster core and thus leads to a systematic underestimate of the Hubble constant.

In this *Letter* we calculate the effect of lensing on SZ measurements of the Hubble constant. Our discussion on lensing follows closely the approach developed in an earlier paper (Refregier & Loeb 1996, hereafter RL) which focused on lensing of the X–ray background by galaxy clusters; the interested reader should consult this earlier paper for more details. Here, we describe our models for the background population of radio sources and for the cluster potential in §2. We then show in §3 how the lensing effect leads to a systematic decrement in the intensity of unresolved sources. In §4, we present numerical results for different values of our model parameters and for the specific example of A2218. Finally, §5 summarizes the main conclusions of this work.

2. Model

We model the gravitational potential of the cluster as a Singular Isothermal Sphere (SIS) (e.g. Schneider et al. 1992). This model provides a good first–order approximation to the projected

mass distribution of known cluster lenses (Tyson & Fischer 1995, Narayan & Bartelmann 1996, Squires et al. 1996a,b). The SIS potential causes background sources to appear brighter but diluted on the sky by the magnification factor

$$\mu(\theta) = \left|1 - \frac{\alpha}{\theta}\right|^{-1}, \tag{1}$$

where θ is the angle between the image of the source and the lens center, and α is the Einstein angle. For a SIS with a line-of-sight velocity dispersion σ_v, $\alpha = 4\pi(\sigma_v^2/c^2)D_{ls}/D_{os}$, where D_{ls} and D_{os} are the lens–source and the observer–source angular diameter distances, respectively.

In general, the Einstein angle α depends on the redshifts of the lens, z_l, and of the source, z_s. However, the dependence on the source redshift is weak if $z_s \gtrsim 3z_l$ (cf. Fig. 1 in RL). Most measurements of the SZ effect are performed with nearby clusters ($z_l \lesssim 0.2$), while sub–mJy radio sources have median redshifts in the range $0.5 \lesssim z_s \lesssim 0.75$ (Windhorst et al. 1993). We therefore take α to be independent of z_s and simply consider the two–dimensional distribution of radio sources on the sky.

We model the flux distribution of background radio sources according to the observed number–flux relation at 4.86 GHz (Windhorst et al. 1993). The squares in Figure 1 show the mean values of the observed differential counts, dn/dS, normalized by S^{-2}. The dotted line illustrates the number counts limits inferred by Fomalont et al. (1991) from a fluctuation analysis of the radio source background (see also Windhorst et al. 1993). The observed counts extend from $\sim 10^{-5}$ to 10 Jy. We model these counts by six broken power laws of the form, $(dn/dS)|_S = \eta_j S^{-\lambda_j}$ for $S_{j-1,j} > S > S_{j,j+1}$, with $j = 1, \ldots, 6$. The six slope parameters are $\lambda_1 = 2.50$, $\lambda_2 = 2.64$, $\lambda_3 = 2.34$, $\lambda_4 = 1.67$, $\lambda_5 = 2.19$, and $\lambda_6 = 1.70$. The power-law break points are $S_{12} = 10$, $S_{23} = 6.0 \times 10^{-1}$, $S_{34} = 2.6 \times 10^{-2}$, $S_{45} = 8.0 \times 10^{-4}$, and $S_{56} = 1.2 \times 10^{-6}$ Jy. The normalization is set by imposing continuity and using $\eta_1 = 54.0$ Jy$^{\lambda_1 - 1}$ sr^{-1}. The model relation is shown as the solid line in figure 1. The counts were extended below 10 μJy based on the extrapolation suggested by Windhorst et al. (1993). For a typical SZ threshold $S_d \gg 10\mu$Jy, the results are not sensitive to the parameters of this extrapolation. Our total 4.86 GHz intensity from radio sources is $i_{tot}(4.86\text{GHz}) \equiv \int_0^\infty S(dn/dS)dS = 3.7 \times 10^3$ Jy sr^{-1} or equivalently 5.1 mK.

At frequencies above 4.86 GHz, we approximate the mean spectrum of the sources by a power law, $S_\nu \propto \nu^{-\gamma}$. Based on the observed fluxes between 1.41–8.44 GHz, Windhorst et al. (1993) estimate the median spectral index of sources with $S_{4.86} \sim 0.1$ mJy to be $\gamma = 0.35 \pm 0.15$. Radio sources with $S_{4.86} \lesssim 1$ mJy have a median angular size $\lesssim 2''$ (cf. Fig. 2 in Windhorst et al. 1993). In the FIRST radio survey, White et al. (1997) found that only $\sim 20\%$ of all sources with $S(1.4\text{GHz}) \gtrsim 1$ mJy have a major axis $\gtrsim 5''$. The angular extent of sub–mJy radio sources is thus much smaller than the Einstein angle of clusters with observed arcs ($\alpha \sim 30''$; see Le Fèvre et al. 1994), as well as the beam size used in SZ measurements ($\gtrsim 1'$; cf. Rephaeli 1995). Thus, we can safely ignore the finite extent of the radio sources and treat them as if they were pointlike.

3. The Lensing Effect

In a region of the sky where the magnification factor is μ, the apparent differential count of sources obtains the value $(dn/dS)|_S = \mu^{-2}\,(d\hat{n}/d\hat{S})\Big|_{S/\mu}$, where hat denotes unlensed quantities. In particular, for a power law differential count relation, $(d\hat{n}/d\hat{S})\Big|_{\hat{S}} \propto \hat{S}^{-\lambda}$, the observed differential count is, $(dn/dS)|_S \propto \mu^{\lambda-2}S^{-\lambda}$. The differential count therefore increases (decreases) as μ increases if λ is above (below) the critical slope $\lambda_{\rm crit} \equiv 2$. When $\lambda = \lambda_{\rm crit}$, lensing has no effect on the apparent differential count. Interestingly, figure 1 shows that the actual radio count slope oscillates around $\lambda_{\rm crit}$ for fluxes $S_{4.86} \lesssim 1$Jy.

In measurements of the SZ effect, discrete sources are typically removed down to a given detection flux threshold, S_d. The mean residual intensity $i(< S_d)$ due to the superposition of all undetected discrete sources with fluxes below S_d is then assumed to be equal to its sky–averaged value

$$\hat{i}(< S_d) = \int_0^{S_d} d\hat{S}\hat{S}\,\frac{d\hat{n}}{d\hat{S}}\bigg|_{\hat{S}}. \tag{2}$$

However, the magnification due to lensing lowers the unresolved intensity *systematically* relative to its sky–averaged value and changes it to,

$$i(< S_d) = \int_0^{S_d} dSS\mu^{-2}\,\frac{d\hat{n}}{d\hat{S}}\bigg|_{S/\mu} = \hat{i}(< S_d/\mu), \tag{3}$$

where $\mu = \mu(\theta)$ is given by equation (1). Lensing conserves the total intensity of the radio source background and merely reduces the effective flux threshold for resolving sources by a factor μ. The intensity offset due to lensing is then, $\Delta i_{\rm lens} \equiv i(< S_d) - \hat{i}(< S_d)$. In the RJ regime, this can be expressed more conveniently in terms of the brightness temperature difference, $\Delta T_{\rm lens} = (c^2/2\nu^2 k_{\rm B})\Delta i_{\rm lens}$, where $k_{\rm B}$ is Boltzmann's constant. For $\mu > 1$ (i.e., $\theta > \alpha/2$), the unresolved intensity is decreased, implying a negative $\Delta T_{\rm lens}$, and so the SZ decrement in the RJ regime, $\Delta T_{\rm SZ}$, is overestimated due to lensing. Note that for $\theta \gg \alpha$, equation (1) yields $\mu \approx 1 + \alpha/\theta$ and $\Delta T_{\rm lens} \propto \theta^{-1}$.

The effect of lensing on estimates of H_0 can be easily found from the scaling, $H_0 \propto (\Delta T_{\rm SZ})^{-2}$, where $\Delta T_{\rm SZ}$ is the temperature offset produced by the SZ effect. The small systematic correction $(\Delta H_0)_{\rm lens} = H_0(\text{true}) - H_0(\text{observed})$, which must be incorporated in order to compensate for the lensing effect is, to leading order,

$$\frac{(\Delta H_0)_{\rm lens}}{H_0} \approx 2\frac{\Delta T_{\rm lens}}{\Delta T_{\rm SZ}}. \tag{4}$$

For $\theta \gtrsim \alpha/2$ and the RJ spectral regime, both $\Delta T_{\rm lens}$ and $\Delta T_{\rm SZ}$ are negative and so $(\Delta H_0)_{\rm lens}$ is positive. The lensing correction will then tend to increase the estimated value of the Hubble constant.

4. Results

Figure 2 shows $\Delta T_{\rm lens}$ as a function of angular separation from the cluster center, θ, for several values of the detection threshold S_d. The values for $\Delta T_{\rm lens}$ and S_d correspond to a frequency of 4.86 GHz. The dependence of $\Delta T_{\rm lens}$ on the Einstein angle α and on frequency ν were conveniently factored out. The Einstein angles for clusters with observed optical arcs are in the range of 10–50$''$ (Le Fèvre et al. 1994).

The lensing decrement, $\Delta T_{\rm lens}$, shows a sharp peak near the Einstein angle. For $\theta \gg \alpha$, $\Delta T_{\rm lens}$ is first weakened and then enhanced as S_d varies from 10^{-2} to 10^{-5} Jy. This is due to the fact that the count slope λ crosses the critical value $\lambda_{\rm crit} = 2$ around $S_d \sim 10^{-3}$ Jy (see Fig. 1). The enhancement in $\Delta T_{\rm lens}$ as S_d decreases below 10^{-3} Jy occurs in spite of the decrease in the unlensed intensity $\hat{i}(< S_d)$ there. At these fluxes, the removal of fainter radio sources paradoxically makes the lensing decrement more pronounced. Note that because of the large shot noise in the source counts (with an rms of $\sigma_i/i \approx 0.5$ in a 1 arcmin^2 cell for $S_d(4.86{\rm GHz}) = 10^{-3}$ Jy), $\Delta T_{\rm lens}$ will *not* necessarily be realized in each individual cluster. The lensing induced decrement should be regarded as a systematic effect that must be corrected for statistically, when a large sample of clusters is considered. For observations with a large field of view, the lensing signature might appear in the outer part of each individual cluster.

As a specific example we consider A2218, an Abell richness class 4 cluster at a redshift $z_l = 0.175$, which shows several optical arcs (Pelló et al. 1992, Le Borgne et al. 1992). Arc no. 359 in Pelló et al. (1992) is separated by $20''.8$ from the central cD galaxy and has a measured redshift of 0.702, close to the probable median redshift of sub-mJy sources ($z_s \sim 0.5$–0.75; cf. Windhorst et al. 1993). We therefore model the cluster potential as a SIS with an Einstein angle of $\alpha = 20''.8$ for our radio sources (see also Miralda-Escudé & Babul 1995). Interferometric imaging of the SZ effect in this cluster was performed by Jones et al. (1993) at 15 GHz, after the removal of point sources with fluxes above $S_d(15{\rm GHz}) \approx 1$ mJy. The restoring beam for their short baseline image had a FWHM of $129'' \times 120''$. The observed angular dependence of $\Delta T_{\rm SZ}$ was fitted by a β–model,

$$\Delta T_{\rm SZ}(\theta) = \Delta T_0 \left(1 + \frac{\theta^2}{\theta_c^2}\right)^{1/2 - 3\beta/2} . \tag{5}$$

Acceptable χ^2 values were obtained for different sets of parameters ranging from $\beta \approx 0.6$, $\theta_c \approx 0'.9$, and $\Delta T_0 \approx 1.1$ mK, to $\beta \approx 1.5$, $\theta_c \approx 2'.0$, and $\Delta T_0 = 0.6$ mK.

Figure 3 shows the expected ratio $\Delta T_{\rm lens}/\Delta T_{\rm SZ}$ for A2218 at 15 GHz, assuming a source spectral index of $\gamma = 0.35$. The ratio is shown for two values of $S_d(4.86{\rm GHz})$ and for the two extreme sets of fit parameters for $\Delta T_{\rm SZ}(\theta)$. The sharp peak at $\theta = 20''.8$ reflects the enhancement in $\Delta T_{\rm lens}$ around the Einstein radius (see Fig. 2). For the $\beta = 1.5$ model, $\Delta T_{\rm lens}/\Delta T_{\rm SZ}$ diverges at $\theta \gtrsim 2'$. If the mass distribution follows the SIS profile, $\Delta T_{\rm lens} \propto \theta^{-1}$ at $\theta \gg \alpha$. Since $\Delta T_{\rm SZ} \propto \theta^{1-3\beta}$ for $\theta \gg \theta_c$ (cf. Eq. [5]), the ratio $(\Delta T_{\rm lens}/\Delta T_{\rm SZ}) \propto \theta^{3\beta-2}$ diverges at large radii if $\beta > 2/3$. The values of β derived from X–ray observations of clusters have a large scatter around a mean value

of ~ 0.65 (Sarazin1988, Jones & Forman 1984, Bahcall & Lubin 1994). Weak lensing studies in the optical band could be used in conjunction with X–ray observations to predict the relative radial behavior of the lensing and SZ decrements in each individual cluster.

It is convenient to average the temperature offset over a circular "top hat" beam of radius θ_b centered on the cluster center, $\langle \Delta T(\theta_b) \rangle \equiv 2\theta_b^{-2} \int_0^{\theta_b} \theta \Delta T(\theta) d\theta$. For the above model of A2218 with $S_d(4.86\mathrm{GHz}) = 10^{-4}$ Jy, the 15 GHz mean temperature offsets due to lensing are $\langle \Delta T_{\rm lens} \rangle \approx -28, -14$, and -1.6 μK, for $\theta_b = 0.4, 1$, and 60 arcmin, respectively. The corresponding decrement ratios are $\langle \Delta T_{\rm lens} \rangle / \langle \Delta T_{\rm SZ} \rangle \approx 0.05, 0.03,$ 0.18, for the $\beta = 1.5$ fit, and $0.03, 0.02, 0.002$ for the $\beta = 0.6$ fit. The fractional correction to the Hubble constant (Eq. [4]) is then $\Delta H_0/H_0 \approx 6$–$10\%, 4$–6%, and 0.4–40% for $\theta_b = 0.4, 1$, and 60 arcmin, respectively, where the ranges reflect the ambiguity in the fit parameters of $\Delta T_{\rm SZ}(\theta)$.

5. Conclusions

We have shown that gravitational lensing of unresolved radio sources leads to a systematic overestimate of the SZ temperature decrement at angles $\theta > \alpha/2$. The amplitude of the lensing effect peaks close to the Einstein angle of the cluster, $\alpha \sim 30''$ (cf. Fig. 2). While $\Delta T_{\rm SZ}$ is independent of frequency in the RJ regime, the lensing decrement $\Delta T_{\rm lens} \propto \nu^{-2-\gamma}$ (with $\gamma \approx 0.35$) is significant only at frequencies $\nu \lesssim 30$ GHz. In clusters where the radial profile of the gas pressure is steeper than that of the dark matter density (e.g., due to a gradient in the gas temperature), the ratio of the lensing to the SZ decrement increases at large projected radii. For observations of A2218 at 15 GHz with a source removal threshold of $S_d(4.86\mathrm{GHz}) = 10^{-4}$ Jy, H_0 could be overestimated by ~ 0.4–40%, for a beam radius in the range of 0.4–60 arcminutes (cf. Fig. 3). The importance of the lensing effect will be enhanced in future observations (including attempts to detect the *kinematic* SZ effect) with greater sensitivity, higher angular resolution, and fainter source removal threshold.

Lensing should also affect the power spectrum of microwave background anisotropies on $\sim 1'$ scales behind the cluster. These anisotropies are expected to originate primarily from the Ostriker–Vishniac effect (Hu & White 1996) and the cumulative SZ effect of other background clusters (Colafrancesco et al. 1994, Rephaeli 1995). Future SZ experiments might be contaminated by noise from these fluctuations ($\Delta T/T \lesssim 10^{-6}$), especially in the outer parts of clusters. However, since these diffuse fluctuations will not be removed, lensing will conserve their net intensity and will not systematically offset the SZ decrement as it does in the case of discrete sources.

We thank D. Helfand for useful comments on the manuscript. This work was supported in part by the NASA grants NAG5-3085 (for AL) and NAGW2507 (for AR).

REFERENCES

Bahcall, N. A., & Lubin, L. M. 1994, ApJ, 426, 513

Birkinshaw, M. 1979, MNRAS, 187, 847

Birkinshaw, M., Hughes, J. P., & Arnaud, K. A. 1991, ApJ, 379, 466

Cavaliere, A., Danese, L., & De Zotti, G. 1977, ApJ, 217, 6

Colafrancesco, S., Mazzotta, P. Rephaeli, Y., & Vittorio, N. 1994, ApJ, 433, 454

Fomalont, E. B., Windhorst, R. A., Kristian, J. A., & Kellermann, K. I. 1991, AJ, 102, 1258

Grainge, K., et al. 1996, MNRAS, 278, L17

Gunn, J. 1978, in Observational Cosmology, ed. A. Maeder, L. Martinet, & G. Tammann (Sauverny: Geneva Observatory), 1

Haehnelt, M. G., & Tegmark, M. 1995, MNRAS, submitted, preprint astro-ph/9507077

Hu, W., & White, M. 1996, A&A, in press, preprint astro-ph/9507060

Jones, C., & Forman, W. 1984, ApJ, 276, 38

Jones et al. 1993, Nature, 365, 320

Le Borgne, J .F., Pelló, R., & Sanahuja, B. 1992, A&AS, 95, 87

Le Fèvre, O., Hammer, F., Angonin, M. C., Gioia, I., M., & Luppino, G. A. 1994, ApJ, 422, L5

Miralda-Escudé, J., & Babul, A. 1995, ApJ, 449, 18

Narayan, R., & Bartelmann, M. 1996, Lectures held at the 1995 Jerusalem Winter School, preprint astro-ph/9606001

Pelló, R., Le Borgne, J. F., Sanahuja, B., Mathez, G., & Fort, B. 1992, A&A, 266, 6

Refregier, A., & Loeb, A. 1996, ApJ, submitted, (RL), preprint available at http://www.astro.columbia.edu∼refreg/publications.html

Rephaeli, Y. 1995, ARA&A, 33, 541

Rephaeli, Y., & Lahav, O. 1991, ApJ, 372, 21

Schneider, P., Ehlers, J., & Falco, E. E. 1992, Gravitational Lenses, (New York: Springer-Verlag)

Sarazin, C. L., 1988, X–ray Emissions from Clusters of Galaxies, (Cambridge: Cambridge Univ. Press)

Silk, J., & White, S.D.M. 1978, ApJ, 226, L103

Squires, G., Kaiser, N., Babul, A., Fahlman, G., Woods, D., Neumann, D., & Böhringer, H. 1996a, ApJ, 461, 572

Squires, G., Kaiser, N., Fahlman, G., Babul, A., & Woods, D. 1996b, ApJ, submitted, preprint astro-ph/9602105

Sunyaev, R. A., & Zel'dovich. Ya., B. 1980, ARA&A, 18, 537

Tyson, J. A., & Fischer, P. 1995, ApJ, 446, L55

Wall, J. V. 1994, Austr. J. Phys., 47, 625

White, R. L., Becker, R. H., Helfand, D. J., & Gregg, M. D. 1997, ApJ, Feb. 1 issue, preprint available at http://sundog.stsci.edu/first/publications.html

Windhorst, R. A., Fomalont, E. B., Partridge, R. B., & Lowenthal, J. D. 1993, ApJ, 405, 498

This preprint was prepared with the AAS LaTeX macros v4.0.

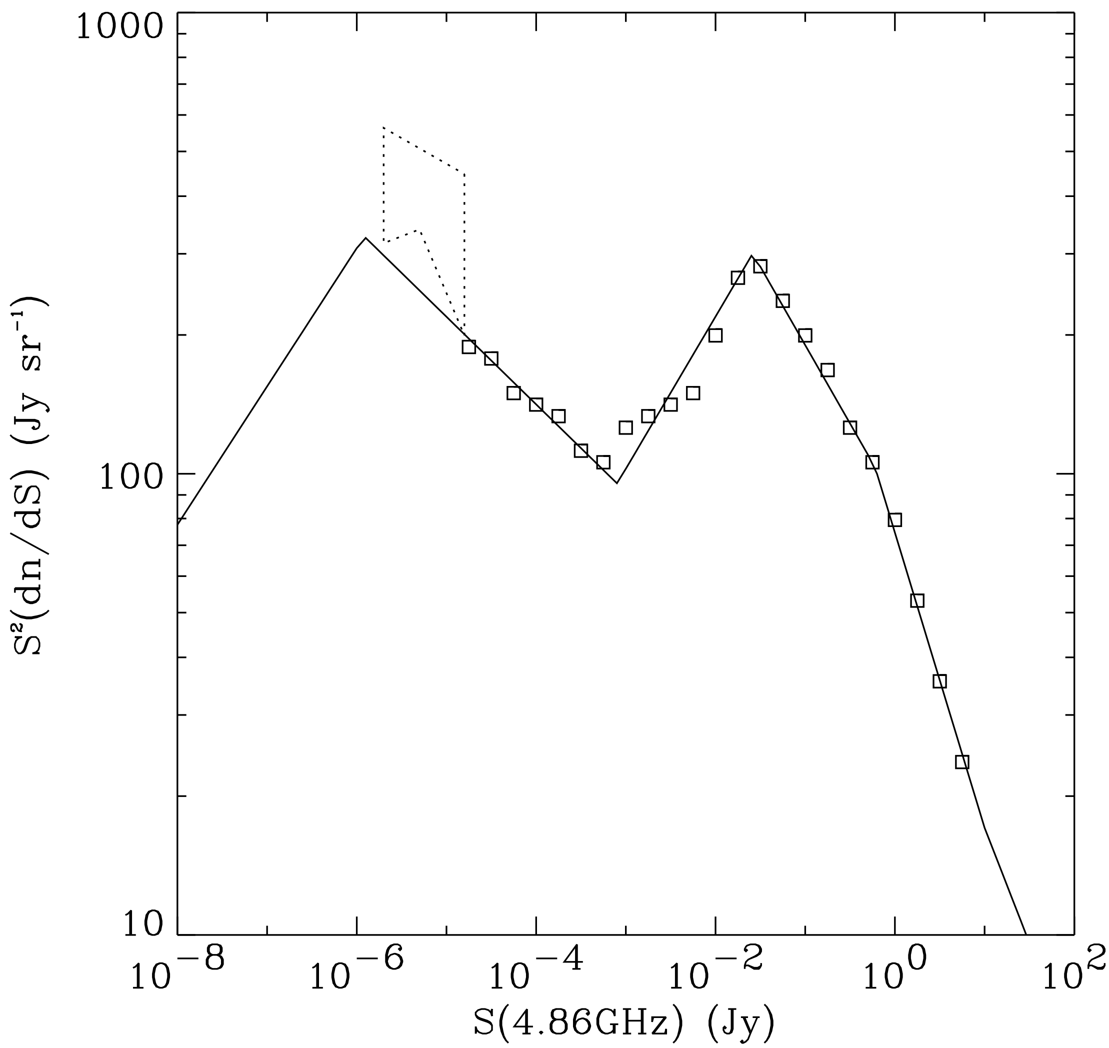

Fig. 1.— Number–flux relation for radio sources at 4.86 GHz. The counts were normalized to S^{-2}, the relation which remains invariant under lensing. The approximate mean counts summarized by Windhorst et al. (1993) are shown as squares. The dotted line corresponds to the limits inferred from a fluctuation analysis of the unresolved background (Fomalont et al. 1991). The solid line shows our model with its six power–law components.

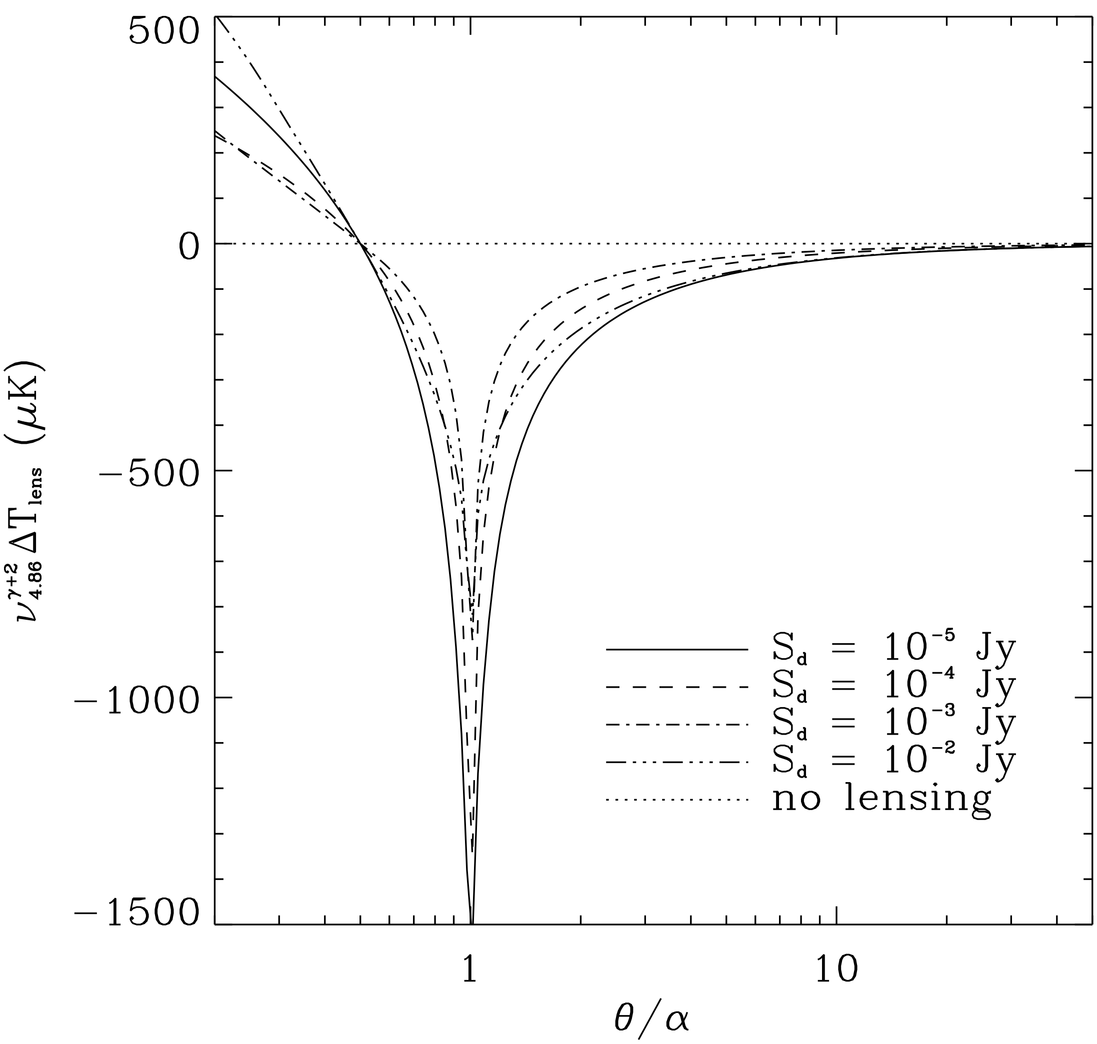

Fig. 2.— Temperature offset $\Delta T_{\rm lens}$ induced by gravitational lensing as a function of angular separation from the cluster center. The offset is shown for several values of the source detection threshold S_d. Values for $\Delta T_{\rm lens}$ and S_d correspond to a frequency $\nu = 4.86$ GHz. The dependence of $\Delta T_{\rm lens}$ on the Einstein angle of the cluster, α, and on frequency, ν, were factored out. The parameter γ is the mean spectral index of the radio sources, and $\nu_{4.86} \equiv \nu/(4.86\text{GHz})$.

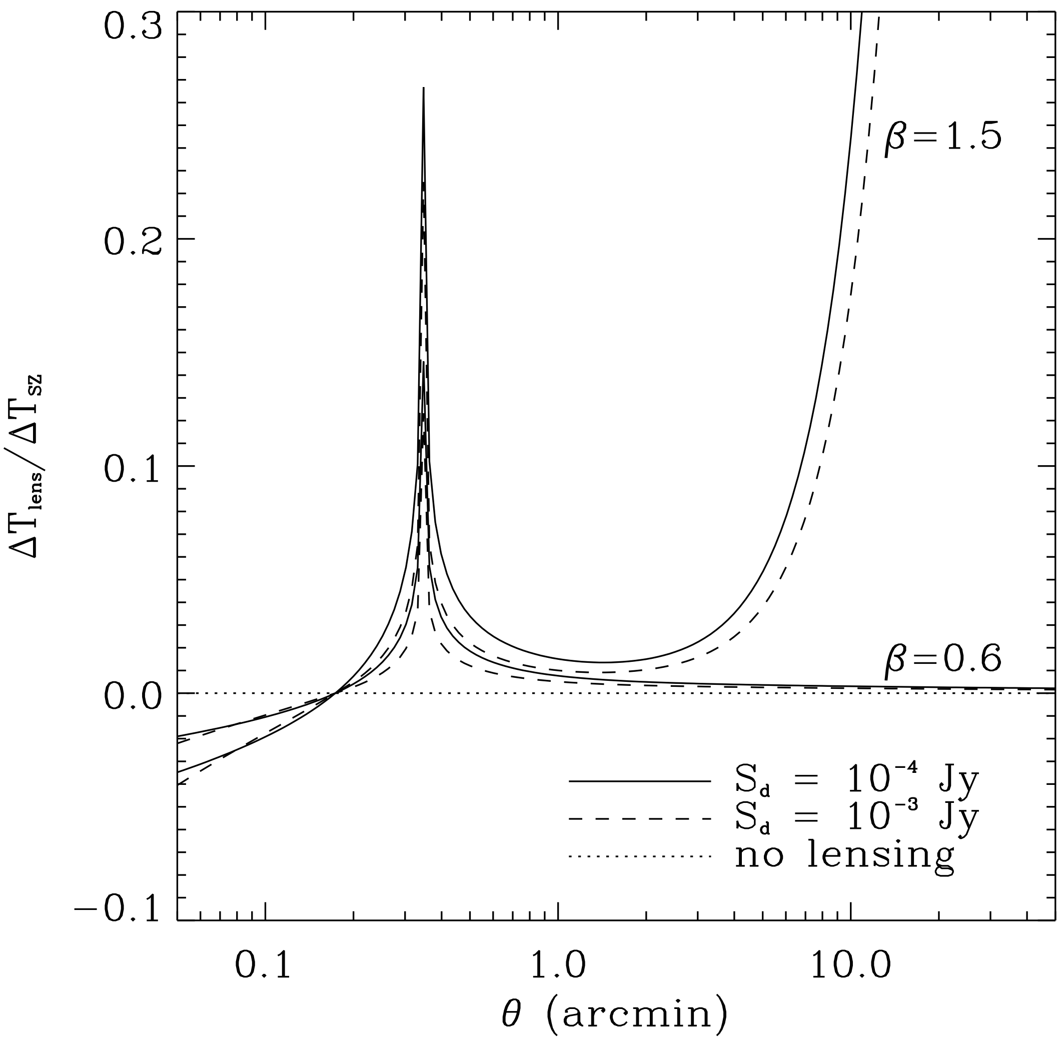

Fig. 3.— Lensing effect in A2218. The ratio of the temperature decrement induced by lensing at 15 GHz, $\Delta T_{\rm lens}$, to that induced by the SZ effect, $\Delta T_{\rm SZ}$, is shown for two values of the flux detection threshold S_d(4.86GHz). The decrement ratio is evaluated for the two extreme fits obtained by Jones et al. (1993) to the observed radial dependence of $\Delta T_{\rm SZ}$. The curves which converge (diverge) at large radii correspond to the $\beta = 0.6$ ($\beta = 1.5$) fit.